

# Forecasting Recessions: Can We Do Better on MARS<sup>(TM)</sup>?

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**M**acroeconomists spend much of their time developing theories and building models to demonstrate how shocks propagate and affect the overall level of economic activity. Both policymakers and the private sector maintain a keen interest in understanding the state of business affairs and the most likely path the economy will take over a planning horizon. Although there are a number of economic events that concern the authorities—including excessive inflation and unemployment—considerable attention is paid to the forecasting of recession. If policymakers can anticipate a recession, they take preemptive corrective action. The private sector uses this information to shelter itself from the vagaries of the business cycle and the most likely reaction of policymakers.

Recently a number of studies have examined the ability of financial variables to forecast recessions. Many analysts find that financial indicators contain information that can be used to increase forecast accuracy. Estrella and Mishkin (1998) found that the slope of the yield curve helped predict recessions beyond one quarter. Haubrich and Dombrosky (1996), Bernard and Gerlach (1996), Dueker (1997), and Atta-Mensah and Tkacz (1998) reported similar results.<sup>1</sup>

Many of these studies employed probit models to estimate the probability of recession. Probit models are sometimes used when economists model the behavior of a dependent variable which takes on two values, e.g., recession = 1, no recession = 0. The traditional approach to probit modeling requires the researcher to choose the variables that will be included in the equation, determine their level of interaction, and assume each variable plays the same role across all recessions in the sample period. These assumptions imply

that the causal nature of recessions remains fixed over time, which we know to be at odds with the stylized facts of American business cycles in the twentieth century.<sup>2</sup> Consequently, probit models may not adequately capture the underlying processes related to recession.

The purpose of this paper is to revisit the information contained in financial variables using nonlinear, nonparametric methods, in particular, multivariate adaptive regression splines (MARS).<sup>3</sup> As with the probit specification, MARS models provide probability forecasts that lie between zero and one, yet they admit a much wider range of possible relationships in the data. The MARS approach allows the series to enter both individually and in combination. Given the idiosyncrasies of the American business cycle, this nonlinear, nonparametric approach may provide greater insight into the factors contributing to recession while avoiding some of the pitfalls associated with the probit specification.

## MODELING WITH MARS

### Data

The National Bureau of Economic Research (NBER) has identified six recessions from January 1960 through September 1999. The dates of these recessions are indicated in the list below. A dichotomous dependent variable that is equal to one if the economy is in recession and equal to zero otherwise will be used as the dependent variable to be forecast.

- April 1960 – February 1961
- December 1969 – November 1970
- November 1973 – March 1975
- January 1980 – July 1980
- July 1981 – November 1982
- July 1990 – March 1991

Recession dates are available at the NBER Web site at <http://www.nber.org>.

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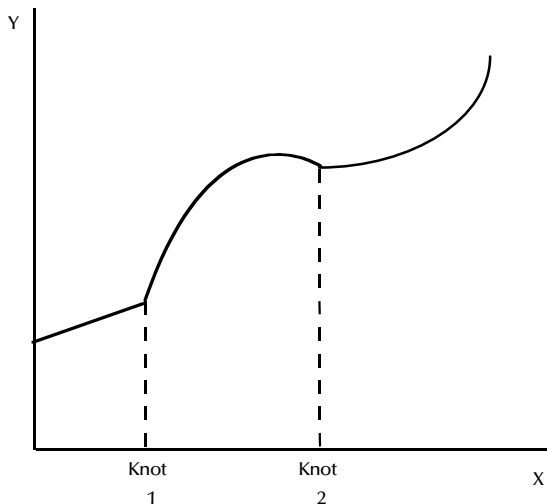
<sup>1</sup> Friedman and Kuttner (1998) report that the closely related paper-bill spread fared less well at predicting the 1990-91 recession. They argue that relative supply conditions in the commercial paper and Treasury Bill markets led to this result. It is worth remembering that although spreads and yield curves contain information on monetary policy, they are a function of returns on assets which are not always perfect substitutes.

<sup>2</sup> See Temin (1998) for an economic historiography of American recessions since 1890.

<sup>3</sup> The professional release of MARS is available from Salford Systems at < <http://www.salford-systems.com> > .

Figure 1

## An Example of Thresholds



A wide variety of financial and real variables have been used as predictors of recession and output growth. The choice of which variables to include depends on whether the analysis is undertaken on monthly or quarterly data. Here the data frequency is monthly, and we employ six variables. The slope of the yield curve (measured by the difference between the 10-year constant maturity Treasury bond rate and the rate on 3-month Treasury bills [secondary market]) has been most prominent in previous studies. Changes in real factors will be captured by the change in the logarithm of the index of industrial production as well as the change in the civilian unemployment rate. Recessions are, after all, persistent declines in real output; thus, past changes in industrial production and the unemployment rate are natural candidates for use as predictors of recessions. The change in the logarithm of the S&P 500 Index has been shown to contain predictive content by Estrella and Mishkin (1998) and Dueker (1997), as have changes in the logarithm of real money, defined to be M2 deflated by the consumer price index. The change in the federal funds rate is also included in the model. These last three variables might capture the effects of both expected and unexpected monetary policy. All series are similar to those examined by others in the literature.

### Nonlinear, Nonparametric Methods

The basic problem facing any forecaster is to determine the fundamental relationship between a

dependent variable,  $Y$ , and a vector of predictors, expressed by  $X$ . The question is how best to specify the functional form  $f\{\cdot\}$  in equation (1):

$$(1) \quad Y = f\{X\} + \varepsilon,$$

where  $\varepsilon$  is the deviation of the dependent variable from the relationship linking  $X$  to  $Y$ . Equation (1) could involve time series on  $X$  and  $Y$ , or cross-sectional data on  $X$  and  $Y$ . The idea behind local nonparametric modeling is to allow for a potentially nonlinear relationship over different ranges of  $X$ .<sup>4</sup>

Friedman (1991a, 1991b) introduced the MARS approach of using smoothing splines to fit the relationship between a set of predictors and a dependent variable. A smoothing spline is similar to a cubic spline, in which a cubic regression<sup>5</sup> is fit to several pre-selected subsets of the data. By requiring the curve segments to be continuous (so that first and second derivatives are non-zero), one obtains a very smooth line that can capture “shifts” in the relationship between variables. These shifts occur at locations designated as “knots” and provide for a smooth transition between “regimes.” The MARS algorithm searches over all possible knot locations, as well as across all variables and all interactions among all variables. It does so through the use of combinations of variables called “basis functions,” which are similar to variable combinations created by using principal components analysis. Once MARS determines the optimal number of basis functions and knot locations, a final least-squares regression provides estimates of the fitted model on the selected basis functions.

As an example, Figure 1 presents the relationship between a single predictor and a dependent variable. This relationship changes at two knot locations—values of  $X_t$  at the points where the relationship between  $X_t$  and  $Y_t$  shifts. We can view

<sup>4</sup> There are a number of attractive nonlinear, nonparametric regression models. Granger (1995) recognized that modeling nonlinear relationships between extended-memory variables holds promise and has recently devised a test for threshold unit roots with Enders (Enders and Granger, 1998). Granger and Terasvirta (1993) examined issues relating to modeling nonlinear relationships in economics. Qi (1999) has demonstrated that stock returns can be predicted accurately using financial and economic variables in a neural network model, whereas Cao and Soofi (1999) and Fernandez-Rodriguez et al. (1999) show that nonlinear methods can be successfully employed in exchange rate prediction.

<sup>5</sup> In a cubic regression between a predictor  $X$  and a dependent variable  $Y$ , the regressors would include a constant, the level of  $X$ , the square of  $X$ , and the cube of  $X$ .

these knots as threshold effects, in that if  $X_t$  is below the first knot (threshold), the relationship appears to be linear. If  $X_t$  is between the two knots, the relationship appears curved; whereas if  $X_t$  is above the second knot, the relationship changes once again. If we label the  $X_t$  variable as time and the  $Y_t$  variable as the price level, Figure 1 tells us something about the behavior of the inflation rate over time. It changes at the knots. A smoothing spline provides a curved transition between the various thresholds exhibited in Figure 1.

When modeling the relationship between a single predictor  $X_t$  and the dependent variable  $Y_t$ , a general model might take the form

$$(2) \quad Y_t = \sum_{k=1}^M a_k B_k(X_t) + \varepsilon_t$$

where  $B_k(X_t)$  is the  $k$ th basis function of  $X_t$ . Basis functions can be highly nonlinear transformations of  $X_t$ , but note that  $Y_t$  is a linear (in the parameters) function of the basis functions. Estimates of the parameters  $a_k$  are chosen by minimizing the sum of squared residuals from equation (2). The advantage of MARS is in its ability to estimate the basis functions so that both the additive and the interactive effects of the predictors are allowed to determine the response variable.

An example will aid in understanding MARS modeling. Suppose the rate of inflation,  $\pi$ , money growth,  $\mu$ , output growth,  $\delta$ , and the rate of currency depreciation,  $\gamma$ , are related according to the following equation:

$$(3) \quad \pi = 1.25 + 0.1 \max(0, \mu - 2.0) + 0.5 \max(0, \delta - 5.0) + 0.8 \max(0, \gamma - 2.5) + 0.25 \max(0, \mu - 2.0) \max(0, \delta - 5.0).$$

The terms in parentheses have effects on inflation only if they are positive and are zero otherwise;  $\max(0, \mu - 2.0)$  is interpreted as the maximum value of the two elements, 0 and  $(\mu - 2.0)$ , and so on. When money growth, output growth, and the rate of currency depreciation are below their threshold values, the inflation rate is 1.25 percent. If money growth is above 2 percent (the value at which there is a knot), this has both direct and indirect (or joint) effects on inflation. The direct effect raises inflation by 0.1 times the difference between money growth and its knot. The joint effect depends on whether output growth is above 5 percent *at the same time that money growth is above its knot or threshold effect*. The rate of currency depreciation raises inflation by 0.8 times the

difference between the rate of currency depreciation and 2.5. Below these knots, for each variable in this example, there are no effects on inflation.

MARS would take money growth, output growth, and the rate of currency depreciation as predictors and attempt to fit the best model for the inflation rate by placing knots and choosing additive and interactive effects to minimize the sum of squared errors. The basis functions would be interpreted as the additive and interactive effects of the variables relative to their knot locations. Thus, in this example, the first basis function would involve  $\max(0, \mu - 2.0)$ ; the second basis function would contain  $\max(0, \delta - 5.0)$ ; the third basis function would be  $\max(0, \gamma - 2.5)$ ; and the final basis function would involve two variables and be nonlinear (in variables):  $\max(0, \mu - 2.0) \max(0, \delta - 5.0)$ .<sup>6</sup>

MARS identifies the knot locations that most reduce the sum of squared residuals. For example, with a single predictor the sum of squared residuals would be

$$(4) \quad \sum_{i=1}^N \left\{ Y_t - \sum_{j=0}^Q b_j X_t^j - \sum_{k=1}^K a_k (X_t - t_k)_+^Q \right\}^2$$

where  $b_j$  and  $a_k$  are multiple regression coefficients on cubic ( $Q = 3$ ) splines of  $X_t$ , and  $X_t$  relative to knot location  $t_k$ . The notation  $(X_t - t_k)_+^Q$  indicates that the cubic spline of  $X_t$  relative to knot location  $t_k$  is included if the difference is positive; otherwise it is zero.

From equation (4) it is clear that the addition of a knot can be viewed as adding the corresponding  $(X_t - t_k)_+^Q$ . A forward and backward stepwise search is incorporated in the MARS algorithm, with the forward step purposely overfitting the data. Insignificant terms are deleted on the backward step of the routine.

Model selection is based on the generalized cross-validation (GCV) criterion of Craven and Wahba (1979). The GCV can be expressed as

$$(5) \quad \text{GVC} = (1/N) \sum_{t=1}^N \left\{ [Y_t - f_M(X_t)]^2 / [1 - C(M)/N]^2 \right\}$$

where there are  $N$  observations, and the numera-

<sup>6</sup> Note that values below these thresholds could be included in the final model if they add to the fit of the equation. For example, one might find a knot in money growth at 1.1, which has a different effect on inflation than that when money growth is above 2.0. Assuming the coefficient to be 0.04, equation (3) would become:

$$\pi = 1.25 + 0.1 \max(0, \mu - 2.0) + 0.5 \max(0, \delta - 5.0) + 0.8 \max(0, \gamma - 2.5) + 0.25 \max(0, \mu - 2.0) \max(0, \delta - 5.0) + 0.04 \max(0, \mu - 1.1).$$

### CAN MARS EXPLAIN M2 VELOCITY SHIFTS?

Because MARS is a relatively new tool in the econometrician's toolkit, an example will help illustrate its potential value. Orphanides and Porter (2000) recently demonstrated how regression trees can be used to explain shifts in M2 velocity, with a view to resurrecting the P\* model of inflation. Regression trees can serve to identify breaks in the reduced-form velocity equation as changes in the coefficient on the opportunity cost of M2 and the time trend. Inflation forecasts based on their estimates of equilibrium velocity outperform those based on the simple Hallman, Porter, and Small (1991) P\* model.

To demonstrate the advantages of using MARS, I constructed estimates of M2 velocity using data identical to those employed by Orphanides and Porter (2000), spanning a somewhat longer time frame, 1959:Q1 to 2000:Q1. Velocity is assumed to be a function of the opportunity cost of M2 balances (the difference between the three-month Treasury bill rate and the average rate paid on M2 balances) and a time trend. MARS allows threshold effects in the opportunity cost and time trend series to accommodate shifts in velocity resulting from financial innovation. Moreover, it allows both series to jointly affect velocity over the sample.

Table A1 provides the final fitted model, allowing as many as 40 basis functions and two variable inter-

actions. The time trend series is most important, whereas the opportunity cost series is only 31.7 percent as important as the trend series. (These figures are constructed on the basis of what happens to the explanatory power of the model when each individual series is excluded from the equation.)

There appear to be threshold effects in the opportunity cost series at 0.27, 2.36, and 3.30 percent, while there are time trend thresholds at observations 20 (1964:Q1), 32 (1967:Q1), 47 (1970:Q4), 58 (1973:Q3), 63 (1974:Q4), 120 (1989:Q1), 130 (1991:Q3), 135 (1992:Q4), and 155 (1997:Q4). Orphanides and Porter (2000) identified time effects at 1960:Q3, 1962:Q3, 1978:Q1, 1988:Q4, and 1992:Q3, as well as a number of interest rate effects spanning from 1.643 percent to 2.034 percent.

MARS provides graphical information on the optimal fit of the data. The surface plot demonstrates that the optimal transformation and combination of both series in explaining M2 velocity is nonlinear.

The actual and fitted MARS model for velocity appear here. As you can see, the fitted MARS model captures velocity shifts in the post-1991 era very well. How this can be used to forecast inflation within a P\* model will be the subject of further work. However, this example demonstrates the potential benefits to MARS modeling.

Figure A1

Surface 1: Maximum = 0.57952

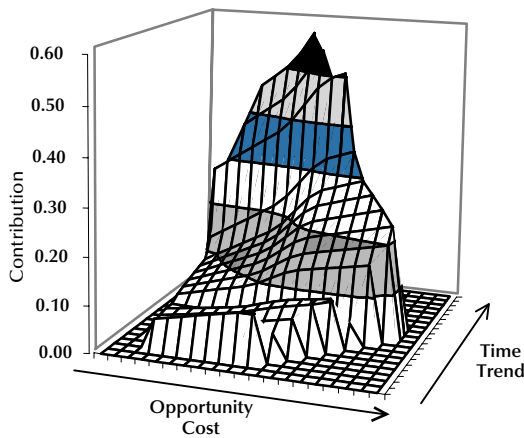


Figure A2

MARS Model for M2 Velocity

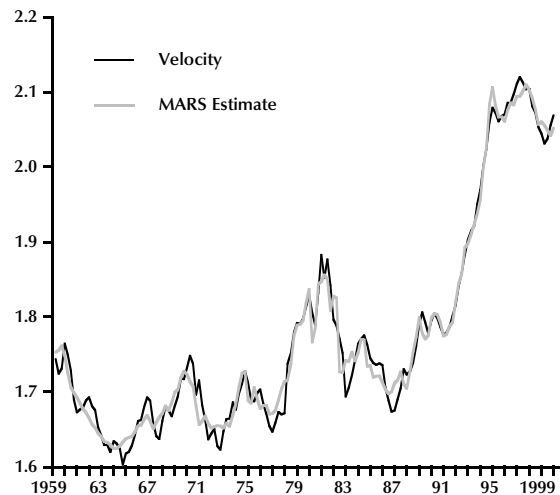


Table A1

## MARS Velocity Estimates

Independent variable	Coefficient	t-Statistic	Variable
Constant	0.982	19.278	
Basis function 2 (BF2)	0.006	13.011	Max (0, 120–time)
Basis function 3 (BF3)	0.018	7.171	Max (0, oppcost–0.270)
Basis function 6 (BF6)	0.008	12.005	Max (0, time–20)
Basis function 8 (BF8)			Max (0, time–155)
Basis function 10 (BF10)	0.007	5.686	Max (0, time–58)* BF3
Basis function 14 (BF14)	–0.002	–5.940	Max (0, time–47)* BF3
Basis function 16 (BF16)	–0.004	–4.481	Max (0, time–63)* BF3
Basis function 18 (BF18)	0.017	7.566	Max (0, time–130)
Basis function 20 (BF20)	–0.017	–7.279	Max (0, time–135)
Basis function 25 (BF25)			Max (0, 3.301–oppcost)
Basis function 27 (BF27)	$0.509 \times 10^{-3}$	3.187	Max (0, 2.364–oppcost)* BF6
Basis function 28 (BF28)	$-0.809 \times 10^{-3}$	–4.168	Max (0, time–32)* BF25
Basis function 40 (BF40)	–0.009	–8.997	Max (0, oppcost–0.270)* BF8
R <sup>2</sup>	0.983		
Number of observations	164		

NOTE: This table provides the final fitted model, allowing as many as 40 basis functions (of which only 11 are retained) and two variable interactions. The time trend series is most important, whereas the opportunity cost series is only 31.7 percent as important as the trend series. These figures are constructed on the basis of what happens to the explanatory power of the model when each individual series is excluded from the equation (*time* denotes the time trend, *oppcost* the opportunity cost series). Also, there appear to be threshold effects in the opportunity cost series at 0.27, 2.364, and 3.301 percent, whereas there are time trend thresholds at observations 20 (1964:Q1), 32 (1967:Q1), 47 (1970:Q4), 58 (1973:Q3), 63 (1974:Q4), 120 (1989:Q1), 130 (1991:Q3), 135 (1992:Q4), and 155 (1997:Q4). Orphanides and Porter (2000) identified time effects at 1960:Q3, 1962:Q3, 1978:Q1, 1988:Q4, and 1992:Q3, as well as a number of interest rate effects spanning from 1.643 percent to 2.034 percent.

tor measures the lack of fit on the  $M$  basis function model  $f_M(X_t)$ . This term corresponds to the sum of squared residuals from the fitted model. The denominator contains a penalty for model complexity,  $C(M)$ , which is related to the number of parameters estimated in the model.

### Interpretation

MARS estimates can most readily be interpreted from an analysis of variance (ANOVA) representation of the model, where the fitted function is expressed as a linear combination of additive basis functions in single variables and interactions between variables. MARS provides graphical plots which illustrate the optimal transformation of the variables chosen by the algorithm, much like the alternating conditional expectations (ACE) algorithm of Breiman and Friedman (1985). The ACE approach to modeling finds the nonlinear transformation of the predictors which maximizes the correlation between the dependent variable and the transformed predictors. A plot of the transformed series against the dependent variable is

sometimes helpful in identifying a functional form to be used in parametric modeling. Hallman (1990) and Granger and Hallman (1991) employed ACE to examine nonlinear cointegration. The accompanying box provides a simple example of the MARS algorithm applied to estimates of M2 velocity.

In MARS, a comparison of the low- and high-order models assists in determining whether to allow variables to enter individually or in combination. Friedman (1991a) suggests a comparison of a measure analogous to an “adjusted  $R$ -squared,” with a model involving interaction terms chosen over an additive model only if its adjusted  $R$ -squared is “substantially” larger. As part of the MARS output, the relative contribution of each variable is determined, as are estimates of the model’s adjusted  $R$ -squared given that a particular ANOVA function (variable) has been omitted from the model. This assists in interpreting the significance of each ANOVA function.

MARS has been extended to incorporate categorical variables, logit regression, and missing data.

It has been successfully applied to the Wolf sunspot data by Lewis and Stevens (1991), to cointegration testing by Sephton (1994), to forecasting exchange rates by Sephton (1993) and De Gooijer et al. (1998), and to nonlinear causality testing by Sephton (1995), as well as in describing large cross-sectional data sets by Steinberg and Colla (1999). The objective here is to examine the extent to which the logit specification provides useful information on the probability of recession.

## FORECASTING RECESSIONS USING MARS

There are two interesting questions to consider. The first relates to in-sample forecasts of the probability of recession based on information that is available at time  $(t - k)$ . That is, how well does MARS fit the historical data? Given the flexibility of the algorithm, one might expect to see MARS perform very well in capturing the probability of recession. The second, more interesting question examines out-of-sample forecasts to determine whether information on financial variables can predict the probability of recession  $k$  periods ahead. This is the type of question one might ask of an “operational forecasting” model: Given data at time  $(t - k)$  how likely is recession within the next few months?

A number of previous studies have examined the ability of probit models to capture recession probabilities.<sup>7</sup> The probit specification examines the probability of recession,  $\text{Prob}(Y_t = 1)$ , using the cumulative standard normal distribution,  $\Phi(\cdot)$  and a set of regressors,  $X_{t-k}$ :

$$(6) \quad \text{Prob}(Y_t = 1) = \Phi(a + \beta X_{t-k})$$

Given data up to period  $(t - k)$  these models are estimated and used to generate recession forecasts at time  $t$ . Statistics on pseudo- $R$ -squared, root-mean-squared error, mean absolute error, and quadratic probability scores are used to gauge forecast accuracy.<sup>8</sup> In-sample forecasts are generally more accurate than out-of-sample forecasts in which an estimated model is used to forecast beyond the estimation period. In the probit model, the parameters are assumed to be temporally stable: that is,  $a$  and  $\beta$  are assumed to be constant. The effects of  $X$  at time  $(t - k)$  are assumed to have the same influence on the probability of recession at every point in the sample. This assumption ignores Temin’s (1998) historiography of American recessions over the past 100 years. Temin concluded

that it was difficult to assign a unique correspondence between an economic variable and the likelihood of recession. The interesting question here is whether MARS results outperform those based on this simple probit specification.

## In-Sample Estimates

For present purposes, recession forecasts are examined at the three-, six-, nine-, and twelve-month horizons. Information available at times  $(t - 3)$ ,  $(t - 6)$ ,  $(t - 9)$ , and  $(t - 12)$  is used to model the probability of recession at time  $t$ . The in-sample evidence is based on estimating a MARS model over the entire sample period. Actual dates of recession are compared with forecasted probabilities to measure the information content of the MARS models.

For example, the six-month horizon model examines the following specification:

$$(7) \quad R_t = f\{Y_{t-6}, \Delta IP_{t-6}, \Delta UR_{t-6}, \Delta RM_{t-6}, \Delta SP_{t-6}, \Delta FF_{t-6}\} + \varepsilon_t$$

where  $R_t$  is 1 if the economy is in recession in period  $t$  and 0 otherwise;  $Y$  is the yield spread,  $\Delta IP$  is the change in the logarithm of industrial production,  $\Delta UR$  is the change in the unemployment rate,  $\Delta RM$  is the change in the logarithm of the CPI deflated value of M2,  $\Delta SP$  denotes the change in the logarithm of the S&P 500 Index, and  $\Delta FF$  denotes the change in the federal funds rate. The error term is given by  $\varepsilon_t$ , with the nonlinear non-parametric functional form given by  $f\{\cdot\}$ . The fitted value can be used to obtain an estimate of the probability of recession given data at time  $(t - 6)$ . Information previous to  $(t - 6)$  and subsequent to  $(t - 6)$  is not included in the model.

The MARS algorithm involves setting a number of parameters used in model selection. The most important are the maximum number of basis functions allowed and the highest order of interaction possible. Because there are six predictors, up to six variable interactions are allowed. The MARS algorithm will fit as many interactions as help

<sup>7</sup> These include Estrella and Mishkin (1998), Haubrich and Dombrosky (1996), Bernard and Gerlach (1996), Dueker (1997), and Atta-Mensah and Tkacz (1998).

<sup>8</sup> The quadratic probability score is simply the average of twice the squared errors. For present purposes the root-mean-squared error and mean absolute error rates will be used to gauge forecast accuracy. Currently there is no measure analogous to the pseudo- $R$ -squared used in probit models.

describe the data, with up to 40 basis functions allowed in the forward search strategy.

Reporting the results of each model would be of little merit because a large volume of output is generated by each estimation. More important is a comparison of the forecast of recession with the actual data. The upper half of Table 1 contains summary statistics on how well each MARS model fit the historical data, as well as those derived from a probit specification using the same explanatory variables. MARS recession probability estimates are superior to those derived from the probit specification, with the root-mean-squared error for the three-month forecasting horizon 16.7 percent relative to 28.9 percent for the probit model. The MARS root-mean-squared error is lowest at the three-month horizon and is highest at the twelve-month horizon, at almost 24 percent. At all horizons the MARS models appear to dominate those based on the probit specification. Figure 2 presents a plot of the MARS probability forecasts for the four different forecasting horizons. The algorithm provides a very good in-sample fit in the short-term, yet exhibits a number of false signals beyond three months.

These results appear to suggest that there are benefits to the modeling of the dichotomous variable at the monthly frequency using MARS. This is to be expected given that nonlinear nonparametric models fare well at explaining relationships in-sample. They are designed to be sufficiently flexible to capture historical data, as are neural network models of Kuan and White (1994). The interesting issue is whether they perform well in an out-of-sample forecasting exercise.

Before turning to that question, it is useful to consider results presented by Dueker (1997). He found that adding a lagged recession variable to the probit framework improved forecast accuracy, arguing that the probability of recession could be affected by duration effects associated with different “states of the world.” Does adding a lagged recession variable affect the in-sample results of both the probit and MARS frameworks?

The bottom half of Table 1 contains information on this augmented model. A recession variable dated at the same time as the other explanatory series was added to the predictor space and MARS models were re-estimated. Forecast accuracy improves at the three-month horizon, with a reduction in the root-mean-squared error from 16.7 to 11.7 percent but remains relatively unchanged at the other time horizons. The probit specification

**Table 1**

In-Sample Forecasting Statistics

Lag	Root-mean-squared error		Mean absolute deviation	
	MARS	Probit	MARS	Probit
<b>Six predictors</b>				
3	.167	.289	.057	.170
6	.197	.298	.078	.177
9	.189	.299	.071	.177
12	.244	.311	.117	.193
<b>Six predictors and lagged dependent variable</b>				
3	.117	.219	.074	.094
6	.199	.280	.158	.154
9	.193	.299	.187	.178
12	.239	.307	.199	.191

NOTE: Root-mean-squared error is calculated by summing the squared differences between the actual and forecast probabilities of recession, dividing by the number of periods in the sample, and taking the square root of the result. The mean absolute deviation is the average absolute value of the prediction less the true state of the recession variable.

**Table 2**

Out-of Sample Forecasting Statistics

Lag	Root-mean-squared error		Mean absolute deviation	
	MARS	Probit	MARS	Probit
<b>Six predictors</b>				
3	.317	.292	.152	.181
6	.311	.293	.145	.160
9	.317	.303	.135	.167
12	.319	.290	.169	.168
<b>Six predictors and lagged dependent variable</b>				
3	.289	.229	.111	.094
6	.299	.281	.129	.135
9	.319	.305	.137	.167
12	.341	.292	.183	.169

NOTE: Root-mean-squared error is calculated by summing the squared differences between the actual and forecast probabilities of recession, dividing by the number of periods in the forecast horizon, and taking the square root of the result. The mean absolute deviation is the average absolute value of the prediction less the true state of the recession variable.

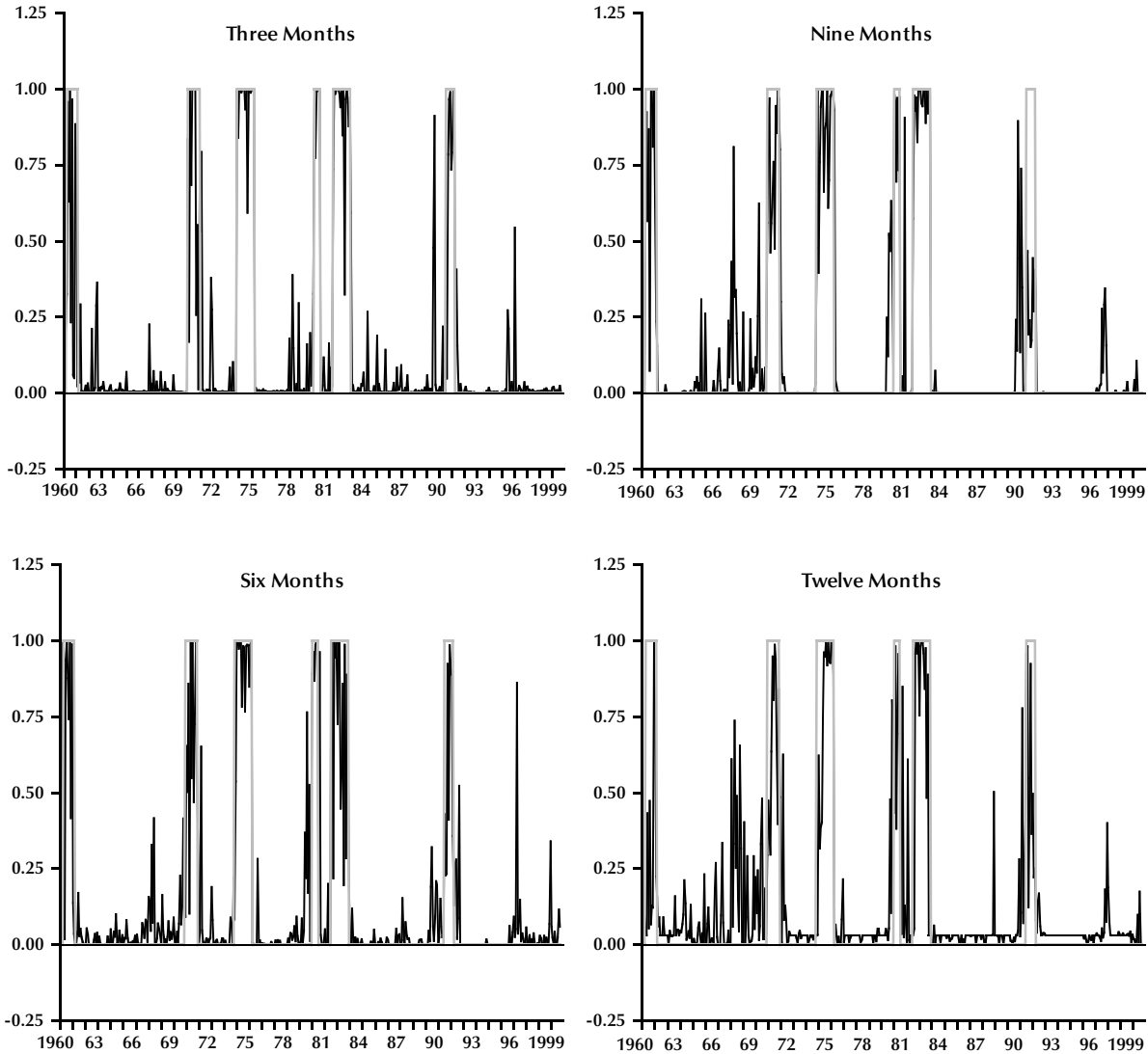
benefits from the addition of the lagged dependent variable at the three-month horizon, but continues to underperform relative to MARS.

**Out-of-Sample Estimates**

Although neural network and nonparametric regression models frequently fit well in-sample, their out-of-sample performance is not as impressive. This is in part a result of the large data samples which are required to fit the models. As well, the models are constructed to provide an optimal

Figure 2

In-Sample Forecasts of Recession, MARS Models



in-sample fit, and, as in traditional linear parametric methods, there is no guarantee they will provide a good fit out-of-sample. A realistic out-of-sample exercise is required to determine whether there are true benefits to modeling recession probabilities using a data-mining procedure such as MARS.

Toward this end, the first 200 observations of the data were used to fit MARS and probit models, which were subsequently used to forecast the probability of recession  $k$  periods hence, with  $k = 3, 6, 9, 12$ , as before.<sup>9</sup> Each forecast was compared with the state of the economy to determine fore-

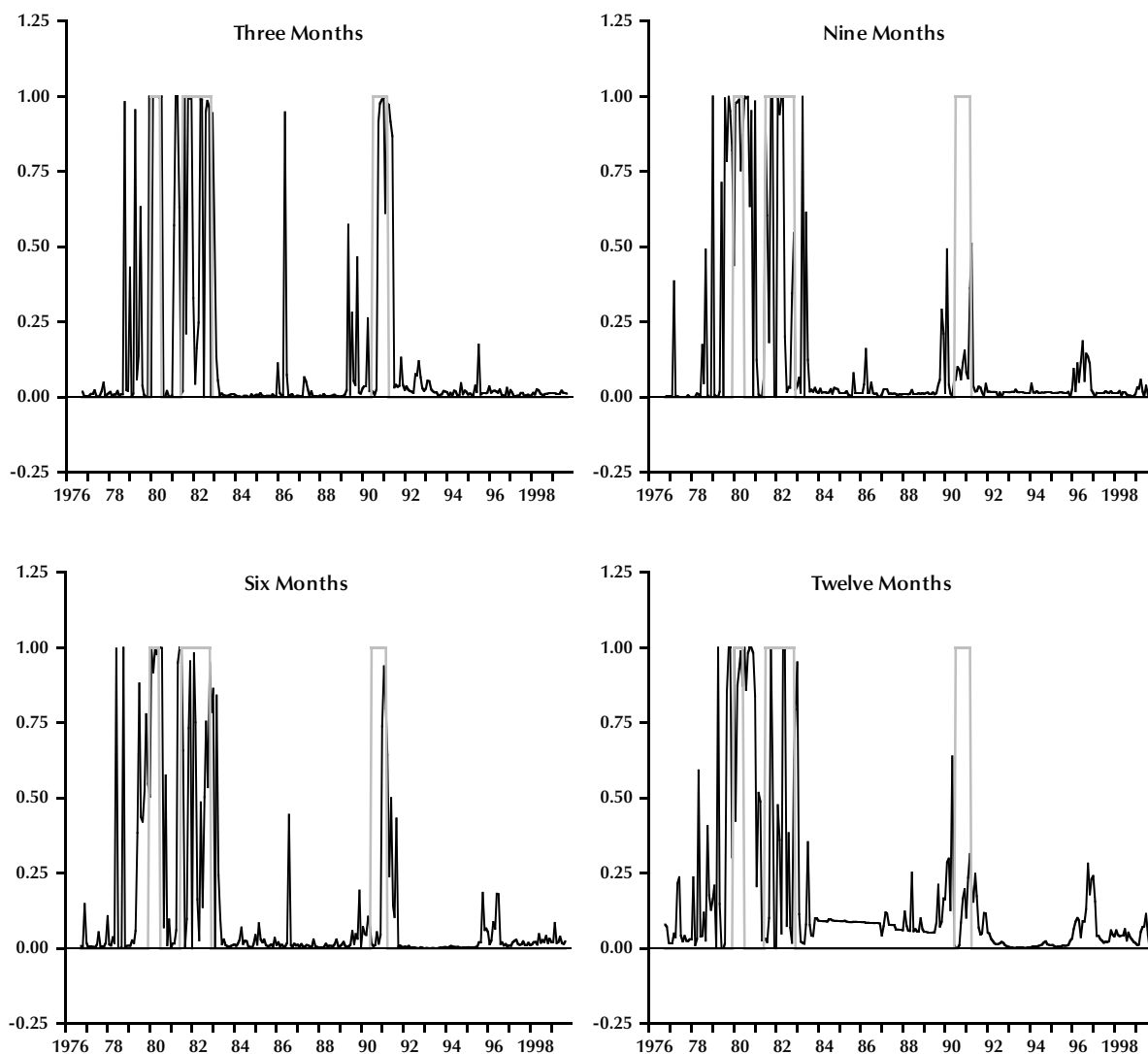
cast accuracy. The sample was then extended by one observation, and the process continued until the entire sample was used to forecast the probability of recession. This process is similar to a rolling regression forecast with model updating.

Table 2 contains summary statistics for the original and the augmented (lagged recession variable) predictor space. The MARS specification does not fare as well at predicting recessions in

<sup>9</sup> The sample used to estimate the first MARS model spans February 1960 through September 1976 and expands by one month until all the data through September 1999 are included.

Figure 3

## Out-of-Sample Forecasts of Recession, MARS Models



out-of-sample forecasting, with root-mean-squared errors around 31 percent using the four different forecast horizons. Adding a lagged recession variable reduces the error rates by less than 3 percent at the three- and six-month horizons, with prediction errors at roughly 29 percent and 30 percent, respectively. The estimated MARS models perform nearly as well as the probit approach to estimating recession probabilities.

Figure 3 presents a plot of the actual recession dates and the MARS forecasts. At the three-month horizon, MARS appears to forecast the 1990-91 recession fairly well, but the large number of false

signals across all time frames suggests that the adoption of nonlinear nonparametric methods is not a panacea for recession forecasting.

## CONCLUSION

For in-sample recession forecasting, the application of multivariate adaptive regression splines to financial predictors of recession shows great promise. The out-of-sample evidence indicates that the MARS models considered here contain helpful, but not entirely accurate, predictions of recession.

There are a number of areas in which the

present analysis can be extended. The first is to include a broader set of financial variables to determine whether they contain information in addition to that already contained in the six series included in the present analysis. Similarly, it may be reasonable to examine these questions using quarterly data rather than monthly data, since the latter may be characterized by a high noise-to-signal ratio.

Finally, the construction of a leading index using the MARS modeling strategy may provide useful forecasts against which to compare other leading indicators maintained by the Conference Board and others. The logit specification may be too difficult for the algorithm to fit effectively; a dynamic model examining economic growth which allows for variable interactions and duration-dependence may offer significant advantages over the present analysis.

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